Equivalent Width Due to Two Overlapping Lines*

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In a recent article Sakai and Stauffer[1] attempt to calculate the equivalent width of two overlapping spectral lines whose line centers do not necessarily coincide. They obtain a very interesting solution to this complex mathematical problem. The purpose of this note is to point out that an exact solution can be obtained for some of the cases considered by these authors and that these solutions have a very different functional form from their results.

Sakai and Stauffer present the results in Fig. 2 for the case when $\nu_{o1} = \nu_{o2}$ and $\gamma_1 = \gamma_2$ (to use their notation). In this case an exact expression for the equivalent width can be obtained by the integration of Eqs. (5), (6), and (7) of reference 1 with the result that

$$W = 2\pi\gamma F(x_1+x_2). \qquad (1)$$

This is to be compared with their approximate result as obtained from Eqs. (7) and (21)

$$W = 2\pi\gamma \left\{ F(x_1) + F(x_2) - \frac{2F(x_1)F(x_2)}{[x_1/F(x_1)]+[x_2/F(x_2)]} \right\} . \qquad (2)$$

The approximate and exact solutions have a very different functional form and the exact solution is considerably simpler! The two expressions agree through terms in the square of x, i.e. when the weak line approximation is valid ($x < 0.2$). However when the strong line approximation is valid ($x > 2$)

(see review in reference 2), Eqs. (1) and (2) have an entirely different functional form. The difference between these two expressions is most significant for $x > 2$. The authors do not compare their approximate expression with the exact equivalent width when either x_1 or x_2 is greater than 10, although such values are of great practical significance.

Thus it might be conjectured that a better expression for the equivalent width of two lines with different positions of their line centers, but with $\gamma_1 = \gamma_2$ is

$$W = 2\pi\gamma D^{-1} F(x_1+x_2), \qquad (3)$$

where

$$D = 1 + \left\{\frac{\nu_{o1}-\nu_{o2}}{[\gamma_1 x_1/F(x_1)]+[\gamma_2 x_2/F(x_2)]}\right\}^2 \qquad (4)$$

is the correction factor introduced by Sakai and Stauffer in their Eq. (21). Since Eq. (3) is correct when $\nu_{o1} = \nu_{o2}$, it is much closer to the correct value for all line separations. Equation (3) reproduces the exact values given in Fig. 3 of reference 1 as closely as this figure can be read.

When $\nu_{o1} = \nu_{o2}$, but $\gamma_1 \neq \gamma_2$, the equivalent width as derived directly from Eqs. (5), (6), and (7) of reference 1 is found to be

$$W = W_1 + W_2 - 2\pi\gamma_1 \frac{2x_1x_2}{1+\xi}, \qquad (5)$$

when the weak line approximation is valid and

$$W = 2\pi\gamma_1(2/\pi)^{\frac{1}{2}} (x_1+\xi^2x_2)^{\frac{1}{2}}, \qquad (6)$$

when the strong line approximation is valid, where

$$\xi = \gamma_2/\gamma_1. \qquad (7)$$

Again Eq. (5) and the weak line limit of Eq. (21) of reference 1 agree. However, when the strong line approximation is valid ($x_1 > 2$; $x_2 > 2$), the results have a very different functional form, since Eq. (21) of reference 1 then reduces to

$$W = 2\pi\gamma_1 \, (2/\pi)^{\frac{1}{2}} \left\{ x_1^{\frac{1}{2}} + \xi x_2^{\frac{1}{2}} - \frac{4}{\pi} \, \frac{x_1^{\frac{1}{2}} \, \xi x_2^{\frac{1}{2}}}{x_1^{\frac{1}{2}} + \xi \, x_2^{\frac{1}{2}}} \right\}. \tag{8}$$

Not only do Eqs. (6) and (8) have a different functional form, but numerical results calculated from them can be quite different whenever $\xi \ll 1$ or $\xi \gg 1$. Thus Eq. (21) of reference 1 should be used with cuation when either line is in the strong line region. A better expression, which is correct in the limit $\nu_{o1} = \nu_{o2}$, for the equivalent width when $x_1 > 2$ and $x_2 > 2$ is

$$W = 2\pi\gamma_1 (2/\pi)^{\frac{1}{2}} \, D^{-1} (x_1 + \xi^2 x_2)^{\frac{1}{2}}, \tag{9}$$

where D is given by Eq. (4).

References

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1 H. Sakai and F. R. Stauffer, J. Opt. Soc. Am. **54**, 759 (1964).

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